Name: RED ID:

Fall 2022 Math 245 Exam 2

Please read the following directions:

Please write legibly, with plenty of white space. Please **print** your name and REDID in the designated spaces above. Please fit your answers into the designated areas; material outside the designated areas (such as on this cover page) will not be graded. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. Each of the ten problems is worth 5-10 points. The use of notes, books, calculators, or other materials on this exam is strictly prohibited, except you may bring one 3"x5" card (both sides) with your handwritten notes. This exam will begin at 10:00 and will end at 10:50; pace yourself accordingly. Good luck!

Special exam instructions for HH-130:

1. Please stow all bags/backpacks/boards at the front of the room. All contraband, except phones, must be stowed in your bag. All phones and smartwatches must be silent, non-vibrating, and either in your pocket or stowed in your bag.
2. Please remain quiet to ensure a good test environment for others.
3. Please keep your exam on your desk; do not lift it up for a better look.
4. If you have a question or need to use the restroom, come to the front. Bring your exam.
5. If you are done and want to submit your exam and leave, please wait until one of the three designated exit times, listed below. Please do **NOT** leave at any other time. If you are sure you are done, just sit and wait until the next exit time, with this cover sheet visible.

Designated exam exit times:
10:20 "I need to work harder"
10:40 "I can't wait to get out of here"
10:50 "I need every second I can get"

REMINDER: Use complete sentences.

Problem 1. Carefully define the following terms:

a. Proof by (vanilla) Induction

b. Big O

Problem 2. Carefully state the following theorems:

a. Proof by Cases theorem

b. Division Algorithm theorem

Problem 3. Prove or disprove: For all $n \in \mathbb{Z}$, we must have $\frac{(n-1)n(n+1)}{3} \in \mathbb{Z}$.

Problem 4. Prove or disprove: For all $x \in \mathbb{R}$, we must have $x\lfloor x \rfloor \leq x\lceil x \rceil$.

Problem 5. Prove or disprove: For all $n \in \mathbb{N}$, we must have $5^n > n^2$.

Problem 6. Solve the recurrence with initial conditions $a_0 = 3, a_1 = -1$ and relation $a_n = a_{n-1} + 6a_{n-2}$ $(n \geq 2)$.

Problem 7. Suppose that an algorithm has runtime specified by recurrence relation $T_n = 9T_{n/3} + n^2$. Determine what, if anything, the Master Theorem tells us.

Problem 8. Let a_n be a sequence of positive real numbers with $\lim_{n\to\infty} a_n = \infty$. Set $b_n = 1 + a_n$. Prove that $a_n = \Theta(b_n)$.

Problem 9. Prove: $\forall x \in \mathbb{R},\ !n \in \mathbb{Z},\ 2n \leq x < 2n + 2$.

Problem 10. Prove: $\forall x \in \mathbb{R},\ \exists n \in \mathbb{Z},\ 2n \leq x < 2n + 2$.